Postscript.

Received June 14, 1871.

After the completion of the above paper, the author found that Mr. Shanks had, in second, third, and fourth supplementary papers on the Constant*, extended his calculations so as to determine γ from $x=2000$, 5000, and 10,000.

The values so obtained all differ in the sixtieth decimal; in fact the higher x is taken, the further from the truth are the results, as the errors in the logarithms are multiplied by larger factors.

The calculation for $x=2000$ affords a verification of the error of log 2; for on subtracting the value of γ $(x=1000)$ from γ $(x=2000)$, we obtain (after correcting B_{13}) a result agreeing with E to 80 decimal places (which is as far as Mr. Shanks has calculated the latter value of γ), with the exception of a difference of a unit in the 73rd figure—an error probably in the summation of the harmonic series from 1000 to 2000.

The values for $x=5000$ and $x=10,000$ are (besides the errors previously noticed) inaccurate from the 62nd figure.

VIII. "Records of the Magnetic Observations at the Kew Observatory. No. IV.—Analysis of the principal Disturbances shown by the Horizontal and Vertical Force Magnetometers of the Kew Observatory from 1859 to 1864." By General Sir Edward Sabine, K.C.B., President. Received June 15, 1871.

(Abstract.)

This paper exhibits an analysis of the principal disturbances recorded by the horizontal and vertical force self-recording magnetometers of the Kew Observatory in the years 1859 to 1864, showing the progressive diminution in the number and value of the disturbances from a maximum in 1859 to a minimum in 1863, being the first moiety of the "decennial period;" and showing also the distribution of the disturbances, increasing or diminishing the respective forces, in the several years, months, and hours.

IX. "Amended Rule for working out Sumner's Method of finding a Ship's Place." By Prof. Sir William Thomson, F.R.S. Received June 15, 1871.

In my previous communication on this subject (*anteà*, p. 259) I described a plan according to which, in the first place, two auxiliary lines were to be drawn on the chart, from two sets of numbers taken out of a proposed Table, and then Sumner's line (the line on which the observation shows the ship to be) was to be interpolated, dividing the space between them in the proportion of the differences of the sun's decli-

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* Proc. Roy. Soc. vol. xvi. pp. 154, 299, vol. xviii. p. 49.

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nation from two of the tabular numbers. I find a better plan in practice to be as follows:—

(1) Take two solutions out of the Table as directed in my previous paper.

(2) Taking the two hour-angles and the two altitudes from these two solutions, interpolate to the nearest minute the hour-angle and the altitude corresponding to the correct declination, according to the simple proportion of its differences from the declinations of the two solutions; and estimate, by inspection, the proper azimuth to the nearest half degree, from the azimuths shown in the two solutions.

(3) Using the interpolated hour-angle, azimuth, and altitude found by clause (2), find on the chart, in the assumed parallel of latitude, the point whose longitude is the difference between the interpolated hour-angle and the Greenwich hour-angle at the time of the observation; through this point draw, by aid of a protractor, a line inclined to the north and south at an angle equal to the azimuth, and on the proper side according to whether the observation was made before or after noon; on this azimuthal line* measure off towards the sun a length (miles for minutes) equal to the correct altitude of observation above the interpolated altitude of clause (2); and through the point thus reached draw a perpendicular to the azimuthal line. This perpendicular is Sumner's line.

The Table (of which a specimen page was shown in my former communication) has now been completed by Mr. Roberts, and has been in my hands long enough to allow me to test its use in actual practice. I find the assistance of compasses for measuring off the assumed colatitude preferable to the slip of card with numbers which I first suggested; and I find the process to be altogether very easy and *unfatiguing* (in respect to fatigue a great contrast to the ordinary method). I find that all the cases (as azimuth and hour-angle both acute, azimuth acute and hour-angle obtuse, or azimuth and hour-angle both obtuse, or, again, declination greater than latitude, but of same name, and declination of opposite name to latitude) work out without ambiguity or perplexity. Still the mere fact of there being different cases may possibly deter practical navigators from leaving the ordinary method, which, though considerably longer and *much more laborious*, has the excellent quality of presenting no variety of cases. I intend, however, to push forward the preparation of a short paper of practical directions, illustrated by examples of all ordinary and critical cases, and to publish it with the Table; so that practical men may have an opportunity of judging from actual experience whether the plan of working Sumner's method which I have proposed will be useful to them or not.

I thought it unnecessary in my former communication to remark that every determination of longitude at sea (except from soundings or sights of land interpreted in connexion with observations for latitude) involves the unknown error of the chronometer, and makes the ship 1′ West or East of the true place for every four seconds of time that the chronometer's indication is in advance of or behind correct Greenwich time. Although I believe that every man who uses a chronometer at sea knows this perfectly well, I shall not omit to state it in the practical directions which I propose to publish, as the Astronomer Royal, Professor Stokes ('Proceedings,' April 27, 1871), and Mr. Gordon (writing in the 'Mercantile and Shipping Gazette') are of opinion that an explicit warning of the kind might be desirable in connexion with any publication tending to bring Sumner's method into more general use than it has been hitherto.

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* It is unnecessary to mark this azimuthal on the chart. By holding one side of a "set square" (or other proper drawing instrument for making right angles) along the azimuthal line, the Sumner line perpendicular to it is readily drawn, and this "Sumner line," or line of equal altitude, is the only mark which need be actually made on the paper.

X. "On Linear Differential Equations."—No. V. By W. H. L. Russell, F.R.S. Received June 15, 1871.

Let us now endeavour to ascertain under what circumstance a linear differential equation admits a solution of the form P $\log_\epsilon$ Q, where P and Q are rational functions of (x).

If $(a_0+a_1x+\ldots.)\frac{d^ny}{dx^n}+(\beta_0+\beta_1x+\ldots.)\frac{d^{n-1}y}{dx^{n-1}}+\ldots.=0,$

we have, substituting $y=\mathrm{P}\log_\epsilon \mathrm{Q}$,

$$\left\{(a_0+a_1x+\ldots.)\frac{d^n\mathrm{P}}{dx^n}+(\beta_0+\beta_1x+\ldots.)\frac{d^{n-1}\mathrm{P}}{dx^{n-1}}+\ldots\right\}\log_\epsilon \mathrm{Q}+\mathrm{R}=0,$$

where R is a rational function of (x). Hence

$$(a_0+a_1x+\ldots.)\frac{d^n\mathrm{P}}{dx^n}+(\beta_0+\beta_1x+\ldots.)\frac{d^{n-1}\mathrm{P}}{dx^{n-1}}+\ldots.=0,$$

or P must be a rational function satisfying the given equation. Having ascertained its value, we have a differential equation of the form

$$\mathrm{L}_0\frac{d^n\log_\epsilon \mathrm{Q}}{dx^n}+\mathrm{L}_1\frac{d^{n-1}\log_\epsilon \mathrm{Q}}{dx^{n-1}}+\ldots.\ \mathrm{L}_n\log_\epsilon \mathrm{Q}=0.$$

Divide this equation by L_n and differentiate, and we have an equation of the form

$$\mathrm{M}_0\frac{d^{n+1}\log_\epsilon \mathrm{Q}}{dx^{n+1}}+\mathrm{M}_1\frac{d^n\log_\epsilon \mathrm{Q}}{dx^n}+\ldots.\ \mathrm{M}_n\frac{d\log_\epsilon \mathrm{Q}}{dx}=0\,;$$

from which we find Q in all possible cases, since $\frac{d\log_\epsilon \mathrm{Q}}{dx}$ is a rational function of (x).

It is impossible that a linear differential equation can in general have a solution of the form $y=f(\log_\epsilon x)$; for in that case we should have

$$(a_0+a_1x+a_2x^2+\ldots.)\frac{d^nf(\log_\epsilon x)}{dx^n}+(\beta_0+\beta_1x+\beta_2x+\ldots.)\frac{d^{n-1}f(\log_\epsilon x)}{dx^{n-1}}+\ldots.=0.$$

Let $z=\log_\epsilon x$, and the equation becomes of the form

$$(a_0+a_1e^z+a_2e^{2z}+\ldots.)\frac{d^nf(z)}{dz^n}+(b_0+b_1e^z+b_2e^{2z}+\ldots.)\frac{d^{n-1}f(z)}{dz^{n-1}}+\ldots.=0;$$